# "APPROACHES OF SOME FIXED-POINT THEOREMS ACCORDING TO FUZZY METRIC SPACE"

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**ABSTRACT**

This study examines the various methodologies utilized for demonstrating fixed-point theorems in the context of metric fuzzy spaces. Fuzzy metric spaces have been found beneficial in analytical mathematics as well as associated disciplines as a result of their versatility in articulating ambiguity and incertitude in real-world problems.

The study begins by introducing the concept of a meter-fuzzy space and elucidating its salient characteristics. The paper then explores the Banach contraction principle while its variations as a landmark result in the field of imprecise metrics and fixed-point theorems in general. We investigate the stability and uniqueness of determined points in fuzzy measure spaces, as well as the conditions under which the Banach (2014) contracting principle may be applied.

In addition, some noteworthy techniques for proving the universality of fixed-point theorems at fuzzy metric areas are presented. that time the aforementioned theorems guarantee the existence of fixed points to feed a collection of mappings defined on an identical space, it is of great importance. The prerequisites and assumptions required for the precise execution of the aforementioned conventional fixed-point propositions are reviewed. It examines the various methodologies employed to demonstrate fixed-point theorems in the framework of fuzzy metric spaces. It has been established that fuzzy measurement spaces are advantageous in analytical mathematics alongside other disciplines because they give an outline to depicting ambiguity as well as uncertainty in real-world problems.

***Keywords: Fixed-point theorems, fuzzy metric space, Banach contraction principle, common fixed-point theorem, fuzzy completeness.***

## 1. INTRODUCTION

Fixed-point formulas play a crucial role in numerous branches of arithmetic while having implications in disciplines as divergent as economics, computing, and construction. These hypotheses demonstrate the existence of points unaffected by specific mappings or transformations. The latest study has focused on stretching classic fixed-point formulas to ambiguous gauge spaces, which serve as an umbrella for modeling inaccuracies and doubt. (Tripathy et al., 2015)

George J. Klir alongside Bo Teng created fuzzy metric spaces in 1995 as an extension of traditional metrics. Fuzzy metric spaces accommodate sets of fuzzy numbers while fuzzy logic allows for a more flexible way to approach position and vicinity. This adaptability is essential for preserving the uncertainty and ambiguity in real-life issues, where accurate estimates are frequently challenging or even impractical. (Tripathy et al., 2015)

In this work, we'll take a look at how different researchers have gone about proving fixed-point theorems in fuzzy metric spaces. The article starts with a high-level introduction to fuzzy metric spaces, focusing on some of their defining features. Fuzzy metrics are proposed as a generalization of the idea of the range that may accurately reflect different degrees of resemblance and indistinguishability among objects.

The application of the Riemann contraction principle to fuzzy metric spaces is the main topic of this study. In classical metric spaces, it is widely known that there are circumstances where a mapping has a single fixed point, a finding described as the Banach contraction theorem. To apply this approach to fuzzy metric spaces, we must think about suitable contraction ideas that take into account fuzziness and imprecision. The stable and convergence features of fixed locations in fuzzy metrics are explored, as are the circumstances under which the Banach constriction principle may be used. (Jain & Jain, 2021)

Basic fixed-point arguments in fuzzy metrical spaces are also investigated. When several mapping is specified in the same space, a common stable-point hypothesis may prove that they all have definitive points at the same time. The paper discusses the presumptions and requirements needed to prove typical fixed-point theorems in fuzzier metric space and unveils various methods and techniques used to prove them. (Jain & Jain, 2021)

Moreover, the importance of fuzzy completeness as a feature of fuzzy metric areas is presented. If a metric space is fuzzy, then any Cauchy succession in that space will also be fuzzy. Fuzzy completeness is examined in connection to fixed-point mathematical theorem with a focus on its function in ensuring both the existence and evolution of definitive points.

## 2. Background and Motivation:

Fixed-point theorems are extensively investigated in traditional measurements and have found numerous applications in numerous mathematical fields along with other fields. These theorems demonstrate the presence of locations that are unaffected by specific maps or modifications, thereby shedding light on the behavior of functions and systems. (Tripathy et al., 2015)

In recent years, curiosity about applying fixed-point equations to fuzzy metrical spaces has increased. Classical metric spaces are generalized by fuzzy metric spaces, which permit a more flexible approach towards a distance or proximity. Problems in the real world frequently involve imprecision, ambiguity, along with uncertainty. They provide a framework of mathematics for modeling and analyzing these types of circumstances.(Imdad & Nafadi, 2014)

Motivating the study in fixed-point theorems to fuzzy metric spaces is a way to overcome the restrictions of conventional metric spaces when tackling dubious or vague facts. Fuzzy metric spaces provide a fitting atmosphere for situations where precision measurements are difficult or impossible.

Studies aim to lay down a scientific basis for analyzing fuzzy systems, fixing optimization issues, or seeing implications for fields including choices, theories of control, computer science, and operational science through the development of methods and techniques to fixed-point proofs in fuzzy metric spaces. (Imdad & Nafadi, 2014)

Mastering fixed-point equations to fuzzy metrical spaces is essential for the advancement of fuzzy mathematics study and techniques. It offers up new possibilities for solving issues with fuzzy logic, fuzzy optimization, and flexible system control, in which unpredictability and ambiguity play an important role.

By looking into tackles to fixed-point the proofs in fuzzy metric spaces, studies can improve their awareness of the attributes and behaviors of descriptions in fuzzy surroundings, resulting in improved techniques to tackle fuzzy optimization problems, developing strong control strategies, and arriving at sound choices regarding circumstances involving unsure or ambiguous knowledge.

In general, the goal of studying fixed-point principles in fuzzy measurement spaces is to expand the relevance of fixed-point hypothesis to the realm of fuzzy along with uncertain data, thereby providing an empirical basis for analyzing and solving challenges in fuzzy settings or releasing novel opportunities for study as well as practical applications.

## 3. Overview of fuzzy metric spaces.

Fuzzy metric areas are a generalization of conventional metric environments that permit a more variable way to approach distance and proximity. In fuzzy mean spaces, a fuzzy vector replaces the standard notion of separation, allowing for the modeling of inaccuracies and doubt.(Kumar & Subhani, 2016)

In a fuzzy measuring space, the gap within two locations is not a discrete real number, instead, it's an uncertain set representing the degree of familiarity or indistinguishability between the points. This fuzzier collection attributes value for membership to varying degrees of proximity, depicting the space's natural fuzziness. Fuzzy metrics generalize classical measurements by allowing for a rise of closeness as opposed to "very near" and "far" dichotomies."(Napit & Nigam, 2023)

A fuzzy metric system is formally defined as a set endowed with a fuzzy statistic that satisfies a set of properties. As in traditional metric spaces, such characteristics include non-negativity, symmetry, personality, or a triangle's inequality. In fuzzy metric places, however, those characteristics have been met concerning a fuzzy sense, signifying the fact that the values that are associated can be sets of fuzzy numbers and not precise integers. (Beg et al., 2023)

Fuzzy metric space is a potent instrument for modeling and analyzing situations characterized by imperfection and uncertainty. It enables a better depiction of difficulties in the actual world in which exact values may be hard or hard to obtain. Fuzzy spaces of metric have implications in numerous fields, which includes fuzzy mathematics, fuzzy optimization, fuzzy systems for control, and uncertainty-based choices. (Sihag et al., 2020)

## 4. Fuzzy Metric Spaces

**Definition and Key Properties:**

A fuzzy measure space can be expressed as the pair (X, d), with X being an array that is not empty and d being a fuzzy statistic on X.

A fuzzy metric d on X is an expression d: X X [0, 1] that meets the following requirements:

Non-negativity: d(x, y) equals 0 for every x, y X, and d(x, y) equals 0 if and only if x = y.

Indiscernible identity: d(x, y) = 1 if just if x = y.

Symmetry: for any x, y X, d(x, y) = d(y, x).

For any x, y, z X, the triangular inequality is d(x, z) mind(x, y), d(y, z).

A fuzzy space of metrics is formed by the set X and the fuzzy metric d. (Kumar & Subhani, 2016)

**Fuzzy Metrics: Generalizing Distances:**

In standard metric spaces, fuzzy dimensions generalize the notion of separation. Distances in traditional metric spaces are non-negative actual numbers that match particular characteristics. Fuzzy vectors reduce these characteristics to make way for varying degrees of element identity or location. The membership values designated by fuzzy metrics reveal the degree of relationship between the parts, with greater similarity leading to higher membership ratings.

Fuzzy metrics offer a more adaptable way to convey proximity, preserving unclear and ambiguous aspect connections. This adaptability is especially useful when addressing real-world problems involving ambiguous or insufficient data. (Sihag et al., 2020)

**Examples of Fuzzy Metric Spaces:**

**Fuzzy Real Line:** Assume the set R of complex s and create the power source fuzzy metric d as follows: R R [0, 1]

d(x, y) = |x - y| / (1 + |x - y|), regardless of x, y ∈ R. That fuzzy metric generalizes the conventional metric upon a true line, permitting progressive changes in values of membership as the distance between units increases.

**Fuzzy Euclidean Space:** Let Rn represent the n-dimensional Riemann space and write the fuzzy gauge d as follows: Rn Rn [0, 1]. the following:

d(x, y) = 1 / (1 + ||x - y||), where |||| symbolises the Geodesic norm and > 0. This ambiguous gauge allocates higher affiliation evaluations to objects that are nearby in Euclidean separation, determining the rate of inclusion value decay.

**Fuzzy Graphs:** On topologies at which the center points signify elements or the borders have a value of membership indicating proximity between vertices, fuzzy spaces for metrics can be determined. ambiguous metrics on graphs enable the capture of ambiguous interactions among network or relational structure elements.

The preceding instances illustrate the range of uses and the relevance of imprecise metric areas for modeling scenarios incorporating imprecision, apprehension, and fuzziness. They lay the groundwork for the study of fuzzy math, the development of fuzzy methods, and the solution of problems in fuzzy optimization, fuzzy systems for control, and fuzzy decision-making.

**Topological fuzzy fixed point theorems**

These theorems show a lack of definitive points for mappings described on the topological areas with fuzzy indices or flexible topologies. They apply the standard fixed-point formulae to fuzzy contexts, accounting for the inaccuracies and unpredictability inherent to fuzzy mathematics. Significant algebraic fuzzy fixed-point theorems are presented below.(Kumar & Subhani, 2016)

**Chang's Fuzzy Fixed-Point Theorem:** The existence of definitive points for fuzz translations based on total metric spaces is established by Chang's theorem. It specifies that T has at minimum one fix point if (X, d) is to total metrics space while T: X X is a fuzzy transformation fitting aware conditions.(Kumar & Subhani, 2016)

**L-fuzzy Fixed-Point Theorem:** This theorem extended the notion of fixed indicates L-fuzzy projections defined on L-fuzzy metric spaces that are complete. It states that T has at least among fixed point if (X, d) is an entire L-fuzzy metric sphere and T: X X is an L-fuzzy transformation satisfying certain conditions.

**Fuzzy Topological Contraction Mapping Principle:** This theorem proves that there are hundreds of bounding points for fuzzy translations constructed with an entire flexible metric space subject to fuzzy geometric contracts. It generalizes the classical Banach contracting Matching Theory to ambiguous environments.(Kumar & Subhani, 2016)

**Fuzzy Topological Fixed-Point Theorem:** Under specific fuzzy topological conditions, this theorem guarantees that there are plenty of bounding points in fuzzy mappings defined with complete curved metric spaces. It establishes the existence of fixed points by combining concepts about fuzzy measurements and flexible topologies.

These geometric fuzzy fixed-point results supply an outline for the exploration of the fixed-point concepts in fuzzy applications allowing for the examination and resolution of issues in fuzzy the subject, fuzzy optimization, and related fields. They exhibit the practicality and benefit of fuzzy calculus in modeling and analyzing uncertain and ambiguous systems, shedding light on the behavior of fuzzy transformations and the reality of definitive locations in fuzzy metric spaces.

"A fuzzy metric space in the sense of George and Veeramani is ordered triple (X, M, ) such that X is a non-empty set, M is a fuzzy set on X × X×[0, ∞)satisfying the following conditions, for all y, z, w X and s, t > 0,

M(y, z, t) > 0;

M(y, z, t) = 1 if and only if y = z;

M(y, z, t) = M(z, y, t);

M(y, z, t) M(z, w, s) ≤ M (y, w, t + s);

M(y, z,): [0, ∞) → [0, 1] is continuous."

## 5. Literature review

**Neelkanth Napit (2023)** This study focuses predominantly on the application of fixed-point principles. Fuzzy topological domains while quasi-fuzzy metric spaces are introduced. Using the outcome of established the concept of variation along with Christi's anchoring point thesis in imprecise geometrical domains, an equation in bounds is derived for Manger's dubious measuring space. In addition to defining a substitute pair of in turn continuum payment transfers, we derive a theorem concerning fixed points using a fuzzy metrics scheme that generates a point that is fixed without requiring the map's contents to be continuous. In addition, V|/-compatible localization can be used in an imprecise measurement setting and the varying radii between points have been determined using controls that differ from previous research. The introduction in R-weakly coincides with type (A,,) and non-compatible mapping in fuzz measurement space verifies a fixed-point theorem not presuming the completeness or continuity of a space. (Nigam, 2023)

**T. RAKESH SINGH (2023)** In this study, the meaning of periodically loosely equal translations and partially equal projected in fuzzy metric space is examined. This paper examines frequent fixed-point results with ambiguous metric space. Using the property of commutativity of coupling pictures, dyad semi-compatible mappings, and inadequately constant rotations that fit the advantageous kind condition for six self-mappings, frequent fixed-point outcomes within fuzzy metric contexts were illustrated. This result strengthens and extends similar research findings. In the area of mathematics, there are numerous issues such as divided feasible challenges, variational imbalance problems, random optimizing issues and homeostasis issues and compatibility issues, and selection and pairing challenges. Steady Point Theory provides the basic tools for problem-solving. (Rakesh, 2023)

**Saif Ur Rehman (2021)**

Using three-self mappings, the present article investigates some coincidental points and common rigid point conclusions in fuzzy metric space. Using the limited connectivity of three-self-mappings, we establish the uniqueness of a few coincidence locations and common uniform point results. To validate our work, we provide concrete instances that demonstrate our findings. Our outcomes improve on and expand a large number of previously published results. In bolstering our studies, we additionally include the utilization of fuzzy linear equations. In the present study, we demonstrate some generalized unique coincidental points and CFP theories for moderately compatible two self-mappings in FM domains without assuming that "fuzzy contractive chains are Cauchy." The "triangular property of FM" is used throughout the entire paper as a fundamental instrument to determine the occurrence of distinct coincidence sites and CFP outcomes in FM spaces. (Rehman, 2021)

**Neena Gupta (2017)**

In this paper, we demonstrate fixed point results in fuzzy measurement space using rational difference. Our findings generalize the findings of numerous other researchers in the realm of science. The primary objective of this paper is to demonstrate fixed-point theorem applications in ambiguous metric space. The initial introduction of fuzzy sets in 1965 lays the groundwork for fuzzy mathematics. The basis here reflects commonplace ambiguity. Several authors afterward created an understanding of fuzzy space by applying different variations of the universal topology of fuzzy sets. The study extends the findings of numerous other authors in the existing literature. Employing the concepts that include a consistent mapping, inferred in this regard, weakly comparable map, or R-weakly consistent map, numerous authors have established some stable point theorems. ( Subhani, 2016)

**Ali Hassan Abbaker Abd Alla (2019)**

In 1965, Zadeh introduced the idea of fuzzy sets in this study. Numerous researchers have developed fuzzy sets in various domains while offering new theories such as fuzzy topology function, fuzzily defined environment, fuzzy metric space, etc. Kamosil while Michalek presented the notion of soft metric space, which is the continuous trapezoidal normed that was established by Schweizer in 1960, in 1975. In this work, we demonstrate the principle of fixed points in fuzzy metric areas and investigate its applicability. In fuzzy metric spaces, we derive a universal stable point thesis in the form of George alongside Veeramani. As a scenario, we as a species prove our hypothesis of integral-type contracting. (Sihag et al., 2020)

**M. Hasan (2023)**

The primary objective of this study is to learn about a few widespread fixed point conclusions of curvilinear mappings for composite pairs of both categories of transformations (single-valued along with multi-valued) in flexible metric-like spaces or exploit the variable distance for its purpose. These theorems generalize and expand upon several earlier findings. In this case, we also utilized a few inferred utility functions to confirm our findings. To confirm the accuracy of our notions and the degree of stimulus generality of our basic result, we as a species offered a real-world example; the plethora of possible uses have contributed to the overall development of fuzzy mathematics. Similar to the investigation of many more concepts, the exploration of fuzzy metrics is currently conducted through a variety of approaches. (Ali Hassan 2019)

**Binod Chandra Tripathy (2015)**

In the year this paper, the author proves periodic site results in the fuzzy metric territory while giving a few standard fixed point formulas for loosely coherent transformations in hazy 3-metric areas beneath a variety of conditions. We demonstrate the three-metrical equivalent of Edelsten's hypothesis in fuzzy 3-metric dimension. In addition, we establish additional outcomes in imprecise 3-metric space. We have developed a periodic value argument in hazy metrical space along with fixed-point hypothesis outcomes for intermittently weakly coherent projections in fuzz 3-metric spaces according to multiple circumstances. Many employees can use these findings for future study and application. (Tripathy et al., 2015)

## 6. DATA COLLECTION

Data collection has become the organized method of gathering and measuring knowledge from sources to obtain complete and accurate information for research purposes. It is not uncommon for bodies while social science investigations, the humanities, and corporations to include a data collection element in their research. It enables scientists along with analysts to capture important factors as data. In contrast to approaches based on the subject matter, the importance of maintaining the correct and truthful order remains constant. The accumulation of current data is essential for maintaining the reliability of research and assuring outstanding outcomes and findings. The results of this investigation will serve as a valuable secondary source for research objectives.

## 7. SECONDARY DATA

Secondary data are the data collected by a third party as opposed to the user. A searcher unaffiliated with an analysis / recherché project accumulates additional knowledge for a different reason, because in the past in very different times, such data were readily easily accessible and economical compared to primary data.

**The main objectives of the study are as follows:**

- Investigation of common fixed point theories in fuzzy metric spaces.
- To investigate popular intuitionistic fixed-point theorems.
- Investigating fixed-point theorems in M-fuzzy metric spaces.

**The hypotheses generated for the present study are as follows.**

H1: There is no significant difference in the common fixed point theorems in fuzzy metric spaces. H2: There is no significant difference in the common fixed point theorems in intuitionistic fuzzy metric spaces.

H3: There is no significant difference in the fixed point theorems in M- fuzzy metric spaces

## 8. Characterization of FA-spaces

Let X be a set, and let F: X P(X) [0, 1] be a fuzzy set fulfilling the requirements FA5 and FA6 such that:

FA5: F(x, A, t) = 1 for all x A X and all t greater than 0. FA6: For every x X and all A P(X), F(x, A, ) = 1. Then, property FA5 may be rewritten as follows:

FA5: For any non-empty subsets A, B X such that A B and A B X, and for every t > 0, we have: F(x, A, t) = 1, which follows F(x, B, t) = 1 for all x X.

In other words, if an element x X has a membership value of 1 in fuzzy set A at time t, it also has a participation value of 1 in fuzzy set B at the same time t, providing that A is a subset of B and both A and B are sets of X.

If an element x holds a substantial amount of membership in a portion of A, then it will likewise have a high amount of involvement in any set B of A, provided that the time component does not change, and this is guaranteed by the characterization of the fuzzy mapping F. A kind of incorporation property is reflected in the fuzzy structure by the fact that it provides a connection amongst the collective attributes of items in related subsets.

Taking into account such fuzzy maps F allows us to investigate the idea of giving X an FA-space (fuzzy approximation space) structure while loosening the criteria FA5 and FA6 compared to the conventional definition. As a result, the fuzzy set attributes and their interactions inside the fuzzy metric frame X may be handled with more flexibility and complexity.

## 9. CONCLUSION

In summary, this study intends to further our knowledge of fixed-point theorems within the setting of fuzzy metric spaces. This paper delivers helpful insight into the theoretical basis and real-world implications of fixed-point principles in the area of fuzzy metric spaces by exploring different methods and techniques, such as the Banach contractions idea, common fixed-point mathematical theorem, and fuzzy completeness. Researchers as well as professionals in disciplines where unpredictability for inaccuracies is natural will benefit from the results given here because they will provide novel instruments for research and problem-solving.

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