Computational Analysis of the Temperature Dependent Viscosity on Ferroconvection in an Porous Medium

[1] Dr. Suresh Govindan, [2]Dr.T.Gayathri
[1,2] Professor, Department of Mathematics, Sri Manakula Vinayagar Engineering College, Puducherry- 605 107, India.

[1]sureshg@smvec.ac.in, [2]gayathrithiiyagu@smvec.ac.in

Abstract **- The effect of temperature dependent viscosity on the threshold of ferroconvective instability in an porous medium using the Brinkman model is studied with computational methods(Galerkin method). It is found that the effect of temperature-dependent viscosity. The presence of porous medium is found to have a destabilizes the system. Numerical computations are made and illustrated graphically.**

Keywords - Porous medium, Brinkman model, Ferroconvection, Galerkin technique , Temperature-dependent viscosity.

I. INTRODUCTION

The study of thermal convection in ordinary fluids saturating a porous medium bounded by rigid and free boundaries have been studied by Patil and Rudraiah (1980) and also by Patil and Vaidyanathan(1981). Thermal conventions of ferrofluids saturating a porous medium has very large application in ferromagnetic fluids entrapped in earth crust. The penetration of ferrofluids in the Hele –Shaw cell was experimentally analysed by Rosensweig et.al.(1978). The stability of magnetic fluid penetration through a porous medium with uniform magnetic field and oblique to the interface have been studied by Zhan and Rosensweig (1980). Walker and Hosmy (1977) have clearly discussed the limit of application of Brinkman and Darcy models.

Sharma(1998) studied thermosolutal instability of Rivlin-Ericksen rotating fluid in porous medium and obtained conditions for non-existence of over stability. The stability of a horizontal fluid and fluid saturated porous layer heated form below has been studied for the case of a time dependent buoyancy force generated by gravity modulation by Malashetty and Padmavathi(1997).

Ramanathan and Suresh (2004) obtained Effect magnetic field dependent viscosity and anisotropy of porous medium on ferroconvection. Soret –driven ferro thermohaline convection by Vaidyanathan et.al(2004). Sekar et.al (2005) studied Effect of sparse distribution pores in a Soret-driven ferro thermohaline convection . Ramanathan and Muchikel (2006) studied effect of Temperature dependent viscosity on ferroconvection in a porous medium. Suresh and Vasanthakumari (2009) analysed Comparison of theoretical and computational ferroconvetion induced by magnetic field dependent viscosity in an anisotropic porous medium.

The present paper focus on the numerical method – Galerkin method in the study of effect of temperature dependent viscosity on ferroconvection in an anisotropic porous medium.

II. MATHEMATICAL FORMULATION

An infinitely spread horizontal layer of Oberbeck-Boussinesq, ferromagnetic fluid of thickness d saturating a sparsely distributed porous medium heated from below is considered. Let ΔT be the temperature difference between the upper and lower boundaries of the fluid. A Cartesian coordinate system is taken with the z-axis vertically upwards.

The fluids viscosity is assumed to be temperature-dependent in the following form (Straughan,2004;Siddheshwar, 2004;Siddheshwar and Cran,2005)

$$\mu(T) = \mu_1\left[1 - \delta(T - T_a)^2\right] \tag{1}$$

where δ is a small positive quantity.

The governing equations used are (Finlayson, 1970; Siddheshwar, 2004)

$$\nabla . \mathbf{q} = 0 \tag{2}$$

$$\rho_0\left(\frac{\partial \mathbf{q}}{\partial t} + (\mathbf{q}.\nabla)\,\mathbf{q}\right) = -\nabla\, p + \rho\,\mathbf{g} + \nabla.(\mathbf{H}.\mathbf{B}) + \nabla.\left[\mu(T)(\nabla\mathbf{q} + \nabla\mathbf{q}^{\mathrm{Tr}})\right] - \frac{\mu(T)}{k_0}\,\mathbf{q} \tag{3}$$

$$\left[\rho_0\, C_{V,H} - \mu_0\mathbf{H}\left[\frac{\partial \mathbf{M}}{\partial \mathrm{T}}\right]_{V,H}\right]\frac{dT}{dt} + \mu_0 T\left[\frac{\partial \mathbf{M}}{\partial T}\right]_{V,H} . \frac{d\mathbf{H}}{dt} = K_1\nabla^2 T \tag{4}$$

$$\rho = \rho_0\left[1 - \alpha\left(T - T_0\right)\right] \tag{5}$$

$$\nabla \times \mathbf{H} = 0\,, \quad \nabla.\mathbf{B} = 0\,, \quad \mathbf{B} = \mu_0(\mathbf{M} + \mathbf{H}) \tag{6}$$

where the anisotropic permeability parameter k_0 takes the value k_1 along xy plane and k_0 takes k_2 value along z axis, also $k_2 = \varepsilon\, k_1$. Superscript Tr in Eq..(3) denotes the transpose

The linearized magnetic equation of state for a single component fluid is

$$M = M_0 + \chi(H - H_0) - K(T - T_a) \tag{7}$$

The magnetic boundary conditions are that the normal component of the magnetic induction and tangential components of the magnetic field are continuous across the boundary. The temperature is assumed constant on each boundary, i.e., T= T_0 at $z = \frac{d}{2}$ and T= T_1 at $z = \frac{-d}{2}$ The basic state is assumed to quiescent. Taking the components of magnetization and magnetic field in the basic state to be $[0,0,M_b(z)]$ and $[0,0,H_b(z)]$, we obtain the following basic state quantities

$$\mathbf{q_b} = 0,\; T_b(z) = T_a - \beta\, z\,,\; p = p_b(z)\,,\; \rho_b(z) = \rho_0\left[1 + \alpha\beta z\right]$$

$$\mu_b(z) = \mu_1\left[1 - \delta\beta^2 z^2\right],\quad \mathbf{H_b} = \left[H_0 - \frac{K\beta\, z}{1+\chi}\right]k\,,\quad \mathbf{M_b} = \left[M_0 - \frac{K\beta\, z}{1+\chi}\right]k \tag{8}$$

where k is the unit vector along the z-axis . In the succeeding section we study the stability of the basic state within the framework of the linear theory.

III. LINEAR STABILITY ANALYSIS

The basic state is disturbed by a small thermal perturbation. Let the components of the perturbed magnetization and the magnetic field be $(M_1{}', M_2{}', M_b(z) + M_3{}')$ and $(H_1{}', H_2{}', H_b(z) + H_3{}')$ respectively.

The perturbed temperature and viscosity are taken to be $T_b(z) + T'$ and $\mu_b(z) + \mu'$ respectively. On linearization, and assuming $K\beta d(1+\chi)H_0$, and using the expressions for $\mathbf{H_b}$ and $\mathbf{M_b}$ in Eq.(8). Eq. (6) and Eq.(7) become

$$H_1{}^1 + M_1{}^1 = \left(1 + \frac{M_0}{H_0}\right)H_i{}^1 \quad (i = 1,2)$$

$$H_3^{\ 1} + M_3^{\ 1} = (1+\chi)H_3^{\ 1} - KT^1 \tag{9}$$

The second of Eq.(6)implies that $H^{'} = \nabla\phi^{'}$, where $\phi^{'}$ is the perturbed magnetic scalar potential. In a further analysis techniques as in Vaidyanathan et.al.(1997) , Ramanathan and Suresh(2004), are used and the vertical component of the momentum equation becomes

$$\rho_0 \frac{\partial}{\partial t}(\nabla^2 w')\mu_b \nabla^4 w' - \frac{\partial^2 \mu_b}{\partial z^2}\left(\nabla_1^{\ 2} w' - \frac{\partial^2 w'}{\partial z}\right) + 2\frac{\partial \mu_b}{\partial z}\nabla^2\left(\frac{\partial w'}{\partial z}\right) + \rho_0 g\alpha \nabla_1^{\ 2} T^1$$
$$- \mu_0 K\beta \frac{\partial}{\partial z}(\nabla_1^{\ 2}\phi^1) + \frac{\mu_0 K^2 \beta}{(1+\chi)}\nabla_1^{\ 2} T^1 - \frac{\mu_b}{k_2}\nabla^2 w' - \frac{1}{k_2}\frac{\partial \mu_b}{\partial z}\left(\frac{\partial w'}{\partial z}\right) \tag{10}$$

where $\nabla_1^{\ 2} = \left(\frac{\partial^2}{\partial x^2}\right) + \left(\frac{\partial^2}{\partial y^2}\right)$ and $\nabla^2 = \nabla_1^{\ 2} + \left(\frac{\partial^2}{\partial z^2}\right)$. The linear form of Eq.(4) in the perturbed state becomes

$$\rho_0 C\frac{\partial T'}{\partial t} - \mu_0 T_a K\frac{\partial}{\partial t}\left(\frac{\partial \phi'}{\partial z}\right) = k_1 \nabla^2 T^1 + \left[\rho_0 C\beta - \frac{\mu_0 K^2 T_a \beta}{1+\chi}\right]w' \tag{11}$$

where $\rho_0 C = \rho_0 C_{V,H} + \mu_0 K H_0$. Using Eq.(9) in the first of Eq.(6), we obtain

$$\left(1+\frac{M_0}{H_0}\right)\nabla_1^{\ 2}\phi' + (1+\chi)\frac{\partial^2 \phi'}{\partial z^2} - K\frac{\partial T'}{\partial z} = 0 \tag{12}$$

IV . NORMAL MODE ANALYSIS

The normal mode solutions of all dynamical variables can be written as

$f(x,y,z,t) = f(z,t)\exp\{i(k_x x + k_y y)\}$

Therefore, one can write

$T^1 = \theta(z,t)\exp\{i(k_x x + k_y y)\}$

$\phi^1 = \phi(z,t)\exp\{i(k_x x + k_y y)\}$ and $w = w(z,t)\exp\{i(k_x x + k_y y)\}$ (13)

where k is the wave number given by

$k^2 = k_x^{\ 2} + k_y^{\ 2}$

Using Eq.(13), Eqs.(10)-(12) become

$$\rho_0 \frac{\partial}{\partial t}\left(\frac{\partial^2}{\partial z^2} - k^2\right)w = \mu_b\left(\frac{\partial^2}{\partial z^2} - k^2\right)w + \frac{\partial^2 \mu_b}{\partial z^2}\left(\frac{\partial^2}{\partial z^2} - k^2\right)w + 2\frac{\partial^2 \mu_b}{\partial z^2}\left(\frac{\partial^2}{\partial z^2} - k^2\right)\left(\frac{\partial w}{\partial z}\right)$$
$$-\alpha\rho_0 g k^2 T + \frac{\mu_0 K^2 \beta}{(1+\chi)}\left[(1+\chi)\frac{\partial \phi}{\partial z} - KT\right]k^2 - \frac{\mu_b}{k_2}\left(\frac{\partial^2}{\partial z^2} - k^2\right)w - \frac{1}{k_2}\frac{\partial \mu_b}{\partial z}\left(\frac{\partial w}{\partial z}\right)$$

(14)

$$\rho_0 C\frac{\partial T}{\partial t} - \mu_0 T_a K\frac{\partial}{\partial t}\left(\frac{\partial \phi}{\partial z}\right) = k_1\left[\frac{\partial^2}{\partial z^2} - k^2\right]T + \left(\rho_0 C\beta - \frac{\mu_0 T_a K^2 \beta}{(1+\chi)}\right)w \tag{15}$$

$$(1+\chi)\frac{\partial^2 \theta}{\partial z^2}\left(1+\frac{M_0}{H_0}\right)k^2\phi - K\frac{\partial T}{\partial z} = 0 \tag{16}$$

The following non-dimensional terms are introduced in our investigation:

$$a = kd, \quad t^* = \frac{\nu t}{d^2}, \quad T^* = \frac{k_1 a R^{\frac{1}{2}}}{\rho_0 C\beta\nu d}T, \quad \phi^* = \frac{(1+\chi)k_1 a R^{\frac{1}{2}}\phi}{K\rho_0 C\beta\nu d^2},$$
$$w^* = \frac{wd}{\nu}, \quad z^* = \frac{z}{d}, \tag{17}$$

where the quantities with asterisks are dimensionless. Using the above non-dimensional terms, Eqs(14) – (16) take the form

$$\left(\frac{\partial}{\partial t}\right)\left(D^2-a^2\right)w=\left(1-Vz^2\right)\left(D^2-a^2\right)w-2V\left(D^2+a^2\right)w-4Vz\left(D^2-a^2\right)Dw+aR^{\frac{1}{2}}\left[M_1 D\phi-(1+M_1)T\right]$$
$$Da\left(1-Vz^2\right)\left(D^2-a^2\right)w+2DaVzDw \tag{18}$$

$$P_r\frac{\partial T}{\partial t}-P_rM_2\frac{\partial}{\partial t}(D\phi)=\left(D^2-a^2\right)T+(1-M_2)aR^{\frac{1}{2}}w \tag{19}$$

$$D^2\phi-a^2M_3\phi-DT=0 \tag{20}$$

where the asterisk have been dropped for simplicity and $D=\frac{\partial}{\partial z}$. The Darcy number $D_a=\frac{d^2}{k_0}$, we recover the system of equations obtained by Finlayson(1970) from Eqns. (18)- (20) when V= 0 and D_a = 0. The typical value of M_2 is 10^{-6} Finlayson (1970) and hence it is assumed negligible. It can be shown , following the analysis of Ramanathan and Suresh(2004), that oscillatory instability does not occur for the problem under consideration. Thus we limit our consideration to stationary instability. The fact that viscosity increases due to the presence of suspended particles supports the contention that oscillatory instability can be discounted in ferromagnetic fluids.

V. NUMERICAL SOLUTION USING GALERKIN METHOD

The boundary conditions are Finlayson(1970)

$w=D^2w=T=D\phi=0$ at $z=\frac{-1}{2}$ and $z=\frac{1}{2}$

using Galerkin technique the power series expansion for the variables are taken as

$w(z,t)=Aw_1(z)e^{i\sigma t}$

$T(z,t)=BT_1(z)\,e^{i\sigma t}$

$\phi(z,t)=C\phi_1(z)e^{i\sigma t}$

using these, we get

$$\{[\sigma-(1-Vz^2)+Da(1-Vz^2)]w_1D^2w_1-[\sigma-(1-Vz^2)+Da(1-Vz^2)]a^2w_1^{\,2}+2Vw_1D^2w_1+$$
$$2Va^2w_1^{\,2}+4Vzw_1D^3w_1-4Vza^2w_1Dw_1-2DaVzw_1Dw_1\}A+aR^{\frac{1}{2}}\left[(1+M_1)\right]w_1T_1B-\left[aR^{\frac{1}{2}}M_1w_1D\phi_1\right]C=0 \tag{21}$$

$$(1-M_2)aR^{\frac{1}{2}}T_1w_1A+\left[\left(P_r\sigma+a^2\right)T_1^{\,2}-T_1D^2T_1\right]B-P_rM_2\sigma T_1D\phi_1C=0 \tag{22}$$

$$-\left[\phi_1DT_1\right]B+\left[\phi_1D^2\phi_1-M_3a^2\phi_1^{\,2}\right]C=0 \tag{23}$$

Taking

$$w(z)=\frac{z^4}{2}-\frac{3z^2}{4}+\frac{5}{32}$$

$$\phi(z)=\frac{z^3}{3}-\frac{z}{4}$$

$$T(z)=z^4+z^2-\frac{5}{16}$$

so as to satisfy the boundary conditions we get the equations

$$\left[\sigma^2+2V-Da-1\right](-0.121428)+(Da-1)V(-0.004166)-\left[\sigma^2+1-Da-2V\right]a^2(0.01230158)-(Da-1)a^2(0.00040593)A+$$
$$aR^{\frac{1}{2}}\left[(1+M_1)\right](-0.025992063)B-\left[aR^{\frac{1}{2}}M_1(-0.02023809)\right]C=0 \tag{24}$$

$$(1-M_2)aR^{\frac{1}{2}}(-0.025992063)A+\left[\left(P_r\sigma+a^2\right)(0.05515873)+(0.56904762)\right]B-$$
$$P_rM_2\sigma(0.04285714)C=0 \tag{25}$$

$$[0.042857]B - [(0.033333) + M_3 a^2 (0.003373)]C = 0 \tag{26}$$

for the existence of non- trivial solutions for the above equations the determinant of the coefficients of A,B and C in (24), (25), and(26) is equated to Zero.
Stationary stability is analyzed using the expression.
Stationary stability is analyzed $\sigma = 0$

$$R = \frac{-x_1 x_5 x_7}{x_3 x_4 x_6 - x_2 x_4 x_7}$$

where, $x_1 = [2V - D_a - 1](-0.121428) + (D_a - 1)V(-0.004166) - [1 - D_a - 2V]a^2(0.01230158) - (D_a - 1)a^2(0.00040593)$

$x_2 = a[(1 + M_1)](-0.025992063)$

$x_3 = aM_1(-0.02023809)$

$x_4 = (1 - M_2)a(-0.025992063)$

$x_5 = [a^2(0.05515873) + (0.56904762)]$

$x_6 = [0.042857]$

$x_7 = -[(0.033333) + M_3 a^2 (0.003373)]$

VI. RESULT AND DISCUSSION

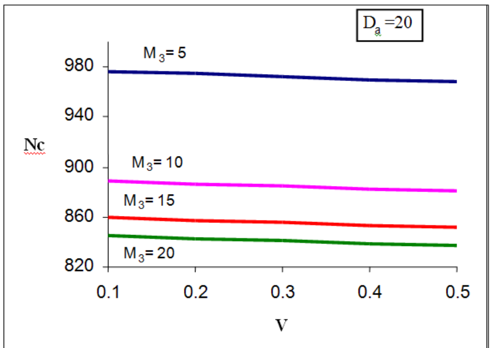

Fig 1: V versus N_c for different values of the magnetization parameter M_3 and ε = 5

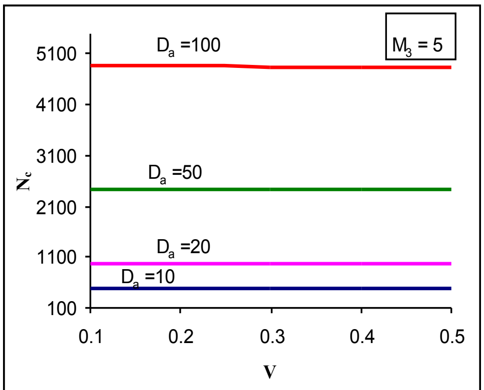

Fig2: V versus N_c for different values of the Darcy Number D_a

The nature and characteristic features of the temperature dependent viscosity are analysed with respect to the above drawn figures

Fig:1 [V Vs N_c] Effect of temperature dependent viscosity for M_3 variation.

Which deals with N_c versus the temperature dependent viscosity parameter V for different values of M_3 with fixed Darcy number $D_a = 20$ proves that the temperature dependent viscosity V exhibits a destabilizing trend.

Fig:2 [V Vs N_c] Effect of temperature dependent viscosity for D_a variation.

Similarly brings out the destabilizing effect of V, when is M_3 is fixed and D_a is varied through the porous matrix shows a stabilizing trend.

VII. CONCLUSION

1. Combining Fig:3 and Fig:4, k_1 variation can be analysed, we conclude that with respect to variation of M_3, ε, D_a, k_1 temperature dependent viscosity V exhibits only a destabilizing effect towards the system.
2. External regulation of ferroconvection is possible in temperature – sensitive liquids.
3. The problem considered in an porous medium ensures that stationary convection is the preferred mode and hence measurements are easy to handle.

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