GRAPHOIDAL TREE COVERING NUMBER OF PRODUCT GRAPHS

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Abstract

A graphoidal tree cover of a graph G is a collection Ŧ oftrees in G such that every path in Ŧ has at least two vertices, every vertex of G is an internal vertex of at most one tree in Ŧ and every edge of G is an exactly one tree in Ŧ. The minimum cardinality of a graphoidal tree cover of G is called the graphoidal tree covering number of G and is denoted by η_T In this paper we determine η_T for some product graphs.

Key words

Graphoidal covers, Acyclic graphoidal cover, Graphoidal tree cover, graphoidal tree d-cover, product graphs.

1 Introduction

A graph is a pair $G = (V, E)$, where$V$ is the set of vertices and E is the set of edges. Here we consider only nontrivial, finite, connected, undirected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively. For graph theoretic terminology we refer to Harary [4].The concept of graphoidal cover was introduced by B.D Acharya and

E. Sampathkumar [1] and the concept of acyclic graphoidal cover was introduced by Arumugam and SureshSuseela [4].The reader may refer [5] and [2] for the terms not defined here.

Let $p=(v_1,v_2,v_3,\ldots,v_r)$ be a path or a cycle in a graph $G=(V,E)$. Then vertices $(v_2,v_3,\ldots,v_{r-1})$ are called internal vertices of P, v_1 and v_r are called external vertices of P. Two paths P and Q of a graph G are said to be internally disjoint if no vertex of G is an internal vertex of both P and Q.

Definition 1.1[1]— A graphoidal cover of a graph G is called a collection ψ of (not necessarily open) paths in G satisfying the following conditions:

(i) Every path in ψ has at least two vertices.

(ii) Every vertex of G is an internal vertex of at most one path in ψ.

(iii) Every edge of G is in exactly one path in ψ.

The minimum cardinality of a graphoidal cover of G is called the graphoidal covering number of G and is denoted by $\eta(G)$.

Definition 1.2[3]—A graphoidal cover ψ of a graph G is called an acyclic graphoidal cover if every member of ψ is an open path. The minimum cardinality of an acyclic graphoidal cover of G is called the acyclic graphoidal covering number of G and is denoted by $\eta_a(G)$ or η_a.

Definition 1.3[4]—A geodesic graphoidal cover of a graph G is a collection ψ of shortest paths in G such that every path in ψ has at least two vertices, every vertex of G is an internal vertex of at most one path in ψ and every edge of G is an exactly one path in ψ. The minimumcardinality of a geodesic graphoidal cover of G is called the geodesic graphoidalcoveringnumber of G and is denoted by η_g.

Definition 1.4[1]—Let ψ be a collection of internally disjoint paths in G. A vertex of G is said to be in the interior of ψ if it is an internal vertex of some path in ψ. Any vertex which is not in the interior of ψ is said to be an exterior vertex of ψ.

Definition 1.5[7] —For any graphoidal cover Ŧ of G, let $t_{Ŧ}$ denote the number of exterior vertices of ψ. Let t=mint_ψ where the minimum is taken over all graphoidal covers of G. Then $\eta=q-p+t$

Corollary 1.6—For any graph G, $\eta \geq q-p$ .Moreover the following are equivalent.

(i) $\eta = q-p$

(ii) There exists a graphoidal cover without exterior vertices.

(iii) There exists a set of internally disjoint and edge disjoint paths without exterior vertices.

In [4] given that $\eta \leq \eta_a \leq \eta_g$ and these inequalities can be strict and also $\eta = \eta_a = \eta_g = n-1$ and Theorem 1.5 and corollary 1.6 are true for geodesic graphoidal covers.

Definition 1.7 [9]

A graphoidal tree cover of a graph G is a collection Ŧof trees in G such that every path in Ŧ has at least two vertices, every vertex of G is an internal vertex of at most one tree in Ŧ and every edge of G is an exactly one tree inŦ. The minimum cardinality of a graphoidal tree cover of G is called the graphoidal tree covering number of G and is denoted by η_T. In this paper we determine η_T for some types graphs.

Definition 1.8 [9]

Let ξ denote the set of all graphoidal tree d-covers of G. Since E(G) is a graphoidal tree d-cover, we have $\xi \neq$. Let $\gamma_T^{(d)}(G) = \min_{I \in \xi} |I|$ .Then $\gamma_T^{(d)}(G)$ is called the graphoidal tree d-covering number of G. Any graphoidal tree d-cover of G for which $|I| \gamma_T^{(d)}(G)$ is called a minimum graphoidal tree d-cover.

Definition 1.9 [9]

A graphoidal tree d-cover $(d \geq 2)$ $\mathscr{J}$ of G is a collection of non-trivial trees in G such that

(i) Every vertex is an internal vertex of at most one tree:

(ii) Every edge is in exactly one tree;

(iii) For every tree $T \in I$, $\Delta(T) \leq d$

Theorem 1.10[9] γ_T $(C_m \times C_n) = 3$ if $m, n \geq 3$

Theorem 1.11[9] $\gamma_T\ (G) \le \left\lceil \frac{p}{2} \right\rceil$ if $\delta(G) \ge \frac{p}{2}$

Definition 1.12

For two graphs G and H, their Cartesian product $G \times H$ has vertex set $V(G) \times V(H)$ in which (g_1, h_1) is joined (g_2, h_2) iff $g_1 = g_2$ and $h_1h_2 \in E(H)$ or $h_1 = h_2$ and $g_1g_2 \in E(G)$

Definition 1.13

For two graphs G and H, their weak product $G \text{o} H$ has vertex set $V(G) \times V(H)$ in which (g_1, h_1) is joined (g_2, h_2) iff $g_1g_2 \in E(G)$ or $h_1 = h_2$ and $h_1h_2 \in E(H)$.

MAIN RESULTS

Theorem 2.1

Let $G = P_5 \times P_5$, then the graphoidal tree covering number of G is $\eta_J(G) = 5$

Proof:

Let $G = P_5 \times P_5$

The graphoidal tree paths are defined as

Let

$V(G) = \{v_{11}, v_{12}, ..., v_{15}, v_{21}, v_{22}, ..., v_{25}, ..., v_{51}, v_{52}, ..., v_{55}\}$

The graphoidal trees as follows

$\Im_1 = \{v_{12}, v_{11}, v_{21}, v_{31}, v_{41}, v_{51}, v_{22}, v_{32}, v_{42}, v_{52}\}$

$\Im_2 = \{v_{13}, v_{12}, v_{22}, v_{32}, v_{42}, v_{52}, v_{23}, v_{33}, v_{43}, v_{53}\}$

$\Im_3 = \{v_{14}, v_{13}, v_{23}, v_{33}, v_{43}, v_{53}, v_{24}, v_{34}, v_{44}, v_{54}\}$

$$\Im_4 = \{v_{15}, v_{14}, v_{24}, v_{34}, v_{44}, v_{54}, v_{25}, v_{35}, v_{45}, v_{55}\}$$

$$\Im_5 = \{v_{15}, v_{25}, v_{35}, v_{45}, v_{55}\}$$

$\therefore$ The graphoidal tree covering number of G is $\eta_J(G) = 5$

Theorem 2.2

Let $G = P_m \times P_n$, then the graphoidal tree covering number of G is $\eta_J(G) = n$

Proof:

Let $G = P_m \times P_n$

The graphoidal tree paths are defined as

Let

$$V(G) = \{v_{11}, v_{12}, ..., v_{1n}, v_{21}, v_{22}, ..., v_{2n}, ..., v_{m1}, v_{m2}, ..., v_{mn}\}$$

The graphoidal trees as follows

$$\Im_1 = \{v_{12}, v_{11}, v_{21}, v_{31}, ..., v_{m1}, v_{m2}\}$$

$$\Im_2 = \{v_{13}, v_{12}, v_{22}, v_{32}, v_{42}, ..., v_{m2}, v_{m3}\}$$

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$$\Im_{n-1} = \{v_{1n}, v_{1n-1}, v_{2n-1}, v_{3n-1}, ..., v_{mn-1}, v_{mn}\}$$

$$\Im_n = \{v_{1n}, v_{2n}, v_{3n}, ..., v_{mn}\}$$

$$G = \Im_1 \cup \Im_2 \cup \Im_3 ... \cup \Im_n$$

$\therefore$ The graphoidal tree covering number of G is $\eta_J(G)=5$

Theorem 2.3

Let $G=P_7 \circ P_7$, then the graphoidal tree covering number of G is $\eta_J(G)=11$

Proof:

Let $G=P_7 \circ P_7$

Let

$V(G)=\{v_{11},v_{12},...,v_{17},v_{21},v_{22},...,v_{27}...,v_{71},v_{72},...,v_{77}\}$

The graphoidal trees coverings are as follows

$\Im_1=\{v_{11},v_{22},v_{33},v_{31},v_{13},v_{42},v_{24}\}$

$\Im_2=\{v_{21},v_{12},v_{32},v_{23}\}$

$\Im_3=\{v_{41},v_{32},v_{23},v_{14},v_{52},v_{43},v_{35},v_{26}\}$

$\Im_4=\{v_{51},v_{42},v_{33},v_{24},v_{15},v_{62},v_{53},v_{44},v_{35},v_{26}\}$

$\Im_5=\{v_{61},v_{52},v_{43},v_{34},v_{25},v_{16},v_{72},v_{63},v_{54},v_{45},v_{36},v_{27}\}$

$\Im_6=\{v_{71},v_{62},v_{53},v_{44},v_{35},v_{26},v_{17},v_{73},v_{64},v_{55},v_{46},v_{37}\}$

$\Im_7=\{v_{72},v_{63},v_{54},v_{45},v_{36},v_{27},v_{74},v_{65},v_{56},v_{47}\}$

$\Im_8=\{v_{73},v_{64},v_{55},v_{46},v_{37},v_{75},v_{66},v_{57}\}$

$\Im_9=\{v_{74},v_{65},v_{57},v_{76},v_{67}\}$

$\Im_{10}=\{v_{75},v_{66},v_{57},v_{77}\}$

$\Im_{11} = \{v_{76}, v_{67}\}$

$\therefore$ The graphoidal tree covering number of G is $\eta_J(G) = 11$

Theorem 2.4

Let $G = P_n \circ P_n$, then the graphoidal tree covering number of G is $\eta_J(G) = 2n-3$

Proof:

Let $G = P_n \circ P_n$

Let

$V(G) = \{v_{11}, v_{12}, ..., v_{1n}, v_{21}, v_{22}, ..., v_{2n}, ..., v_{n1}, v_{n2}, ..., v_{nn}\}$

The graphoidal trees coverings are as follows

$$\Im_1 = \left\{v_{11}, v_{22}, v_{\frac{n-1}{2},\frac{n-1}{2}}, v_{\frac{n-1}{2}1}, ..., v_{1\frac{n-1}{2}}, v_{\frac{n-1}{2}+1,2}, v_{2,\frac{n-1}{2}+1}\right\}$$

$$\Im_2 = \left\{v_{21}, v_{12}, ..., v_{\frac{n-1}{2},2}, v_{2,\frac{n-1}{2}}\right\}$$

$$\Im_3 = \left\{v_{\frac{n-1}{2}+1,1}, v_{\frac{n-1}{2},2}, v_{2,\frac{n-1}{2}}, v_{1,\frac{n-1}{2}+1}, ..., v_{n-2,2}, v_{\frac{n-1}{2}+1,\frac{n-1}{2}}, v_{\frac{n-1}{2},n-2}, v_{2,n-1}\right\}$$

$$\Im_4 = \left\{v_{n-2,1}, v_{\frac{n-1}{2}+1,2}, v_{\frac{n-1}{2},\frac{n-1}{2}}, v_{2,\frac{n-1}{2}+1}, v_{1,n-2}, ..., v_{n-1,2}, v_{n-2,\frac{n-1}{2}}, v_{\frac{n-1}{2}+1,\frac{n-1}{2}+1}, v_{\frac{n-1}{2},n-2}, v_{2,n-1}\right\}$$

$$\Im_5 = \left\{v_{n-1,1}, v_{n-2,2}, v_{\frac{n-1}{2}+1,\frac{n-1}{2}}, v_{\frac{n-1}{2},\frac{n-1}{2}+1}, v_{2,n-2}, v_{1,n-1}, ..., v_{n,2}, v_{n-1,\frac{n-1}{2}}, v_{n-2,\frac{n-1}{2}+1}, v_{\frac{n-1}{2}+1,n-2}, v_{\frac{n-1}{2},n-1}, v_{2,n}\right\}$$

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$\Im_{2n-5} = \{v_{n,4}, v_{n-1,n-2}, v_{n-2,n}, ..., v_{n,n-1}, v_{n-1,n}\}$

$\Im_{2n-4} = \{v_{n,n-2}, v_{n-1,n-1}, ..., v_{n-2,n}, v_{n,n}\}$

$\Im_{2n-3} = \{v_{n,n-1}, ..., v_{n-1,n}\}$

$\Im_{n-1} = \{v_{1n}, v_{1n-1}, v_{2n-1}, v_{3n-1}, ..., v_{mn-1}, v_{mn}\}$

$\Im_{n} = \{v_{1n}, v_{2n}, v_{3n}, ..., v_{mn}\}$

$G = \Im_1 \cup \Im_2 \cup \Im_3 ... \cup \Im_n$

$\therefore$ The graphoidal tree covering number of G is $\eta_J(G) = 2n-3$

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