GRAPHOIDAL TREE COVERING NUMBER OF GRAPHS

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Abstract

A graphoidaltree cover of a graph G is a collection Ŧ oftrees in G such that every path in Ŧ has at least two vertices, every vertex of G is an internal vertex of at most one tree in Ŧ and every edge of G is an exactly one tree in Ŧ. The minimum cardinality of a graphoidal treecover of G is called the graphoidaltree covering number of G and is denoted by η_T In this paper we determine η_T for some types graphs.

Key words

Graphoidal covers, acyclicgraphoidal cover, Graphoidal treecover, bicyclic graph,

A one – point union of two cycles *U(l;m),* A long dumbbell graph *D(l,m,i),* A cycle with a long chord $C_m(i;l)$, graphoidal tree d-cover, graphoidal tree cover.

1 Introduction

A graph is a pair $G=(V,E)$, where$V$ is the set of vertices and E is the set of edges. Here we consider only nontrivial, finite, connected, undirected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively. For graph theoretic terminology we refer to Harary [4].The concept of graphoidal cover was introduced by B.D Acharya and

E. Sampathkumar [1] and the concept of acyclic graphoidal cover was introduced by Arumugam and Suresh Suseela [4].The reader may refer [5] and [2] for the terms not defined here.

Let $p=(v_1,v_2,v_3,\ldots,v_r)$ be a path or a cycle in a graph $G=(V,E)$. Then vertices $(v_2,v_3,\ldots,v_{r-1})$ are called internal vertices of P and v_1 and v_r are called external vertices of *P*. Two paths *P* and *Q* of a graph G are said to be internally disjoint if no vertex of *G* is an internal vertex of both P and *Q*.

Definition 1.1[1]

A graphoidal cover of a graph G is called a collection Ψ of (not necessarily open) paths in G satisfying the following conditions:

(i) Every path in Ψhas at least two vertices.

(ii) Every vertex of G is an internal vertex of at most one path in Ψ.

(iii) Every edge of G is in exactly one path inΨ.

The minimum cardinality of a graphoidal cover of G is called the graphoidal covering number of G and is denoted by $\eta(G)$.

Definition 1.2[3]

A graphoidal cover Ŧ of a graph G is called an acyclic graphoidal cover if every member ofŦ is an open path. The minimum cardinality of an acyclic graphoidal cover of G is called the acyclic graphoidal covering number of G and is denoted by $\eta_a(G)$or η_a .

Definition 1.3[4]

A geodesic graphoidal cover of a graph G is a collection Ŧ of shortest paths in G such that every path in Ŧ has at least two vertices, every vertex of G is an internal vertex of at most one path in Ŧ and every edge of G is an exactly one path in Ŧ. The minimumcardinality of a geodesic graphoidal cover of G is called the geodesic graphoidal coveringnumber of G and is denoted by η_g .

Definition 1.4[1]

Let Ŧ be a collection of internally disjoint paths in G. A vertex of G is said to be in the interior of Ŧ if it is an internal vertex of some path in Ŧ. Any vertex which is not in the interior of Ŧ is said to be an exterior vertex of Ŧ.

Definition 1.5[1]

For anygraphoidal cover Ŧ of G, let $t_{Ŧ}$ denote the number of exterior vertices of Ŧ. Let t=min$t_{Ŧ}$ where the minimum is taken over all graphoidal covers of G. Then $\eta = q - p + t$

Corollary 1.6

For any graph G, $\eta \geq q - p$ .Morever the following are equivalent.

(i) $\eta = q - p$

(ii) There exists a graphoidal cover without exterior vertices.

(iii) There exists a set of internally disjoint and edge disjoint paths without exterior vertices.

In [4] given that $\eta \leq \eta_a \leq \eta_g$ and these inequalities can be strict and also $\eta = \eta_a = \eta_g = n-1$ and Theorem 1.5 and corollary 1.6 are true for geodesic graphoidal covers.

They observe that $\eta_g = q$ if and only if G is Complete. Further for a cycle C_m ,

$$\eta_g = \begin{cases} 2 & \text{if m is even} \\ 3 & \text{if m is odd} \end{cases}$$

Definition 1.7 [8]

A connected (*p*, *p*+1) graph *G* is called a bicyclic graph.

Definition 1.8[8]

A one – point union of two cycles is a simple graph obtained from two cycles, say C_l and C_m where $l, m \geq 3$, by identifying one and the same vertex from both cycles. Without loss of generality, we may assume the *l*-cycle to be $u_0u_1\ldots u_{l-1}u_0$ and the *m*-cycle to be $u_0u_lu_{l+1}\ldots u_{m+l-2}u_0$. We denote this graph by *U(l;m)*

Definition 1.9[8]

A long dumbbell graph is a simple graph obtained by joining two cycles $C_l$andC_m where $l,m \geq$ 3,with a path of length i , $i \geq 1$. Without loss of generality, we may assume $C_l = u_0u_1\ldots u_{l-1}u_0$, $P_i = u_{l-1}u_lu_{l+1}\ldots u_{l+i-1}$ and $C_m = u_{l+i-1}u_{l+i}\ldots u_{l+m+i-2}u_{l+i-1}$.We denote this graph by *D(l,m,i)*

Definition 1.10 [8]

A cycle with a long chord is a simple graph obtained from an *m*-cycle, $m \geq 4$, by adding a chord of length l where $l \geq 1$. Let the *m*-cycle be $u_0u_1\ldots u_{m-1}u_0$. Without loss of generality, we may assume the chord joins u_0 with u_i, where $2 \leq i \leq m-2$. That is ,$u_0u_mu_{m+1}\ldots u_{l+m-2}u_i$ is the chord. We denote this graph by $C_m(i;l)$

Definition 1.11 [9]

A graphoidal tree cover of a graph G is a collection Ŧof trees in G such that every path in Ŧ has at least two vertices, every vertex of G is an internal vertex of at most one tree in Ŧ and every edge of G is an exactly one tree in Ŧ. The minimum cardinality of a graphoidal tree cover of G is called the graphoidal tree covering number of G and is denoted by η_T. In this paper we determine η_T for some types graphs.

Definition 1.12 [9]

Let ξ denote the set of all graphoidal tree d-covers of G. Since E(G) is a graphoidal tree d-cover, we have $\xi \neq$. Let $\gamma_T^{(d)}(G) = \min_{I \in \xi} |I|$ .Then $\gamma_T^{(d)}(G)$ is called the graphoidal tree d-covering number of G. Any graphoidal tree d-cover of G for which $|I| \gamma_T^{(d)}(G)$ is called a minimum graphoidal tree d-cover.

Definition 1.13 [9]

A graphoidal tree d-cover $(d \geq 2)$ of G is a collection of non-trivial trees in G such that

(i) Every vertex is an internal vertex of at most one tree:

(ii) Every edge is in exactly one tree;

(iii) For every tree $T \in \mathrm{I}$ $,\Delta(T) \le d$

Definition 1.14

A flower (C3, Fn) is a graph formed by taking one copy of C3 and 3 copies of Fn and grafting the (v_1, x) XE(Fn) in each edge of C3. A flower (C3, Fn) is denoted by FC3, Fn . Let n be a positive integer with n ≥ 3. We define the vertex set and the edge set of FC3, Fn as follows.

Theorem 1.15 [9] $\gamma_T\ (K_p) = \left\lceil \frac{p}{2} \right\rceil$

Theorem 1.16 [9] $\gamma_T\ (K_{n,n}) = \left\lceil \frac{2n}{3} \right\rceil$

Theorem 1.17 [9] If m ≤ n < 2m - 3, then $\gamma_T\ (K_{n,n}) = \left\lceil \frac{m+n}{3} \right\rceil$.

Further more *if* n > 2m - 3, then $\gamma_T\ (K_{n,n}) = m$

Theorem 1.18 [9] $\gamma_T\ (C_m \times C_n) = 3 \text{if } m,n \ge 3$

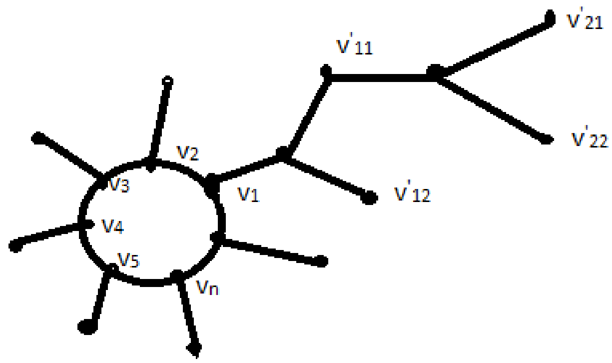
Theorem 1.19 [9] $\gamma_T\ (G) \le \left\lceil \frac{p}{2} \right\rceil \text{if } \delta(G) \ge \frac{p}{2}$

MAIN RESULTS

Theorem 2.1

Let G be aunicyclic graph with unique cycle C . Let n denote the number of pendant vertices of G. Then,the graphoidal tree covering number of G is $\eta_T(G)=2$

Proof:



Let $G=V(l,m)$

$V(G)=\{V_1,V_2,...,V_n,V_{i1}',V_{i2}',...,V_{in}',V_{i+1,1}',V_{i+1,2}',...,V_{n_1}',V_{n_2}',...\}$

$P_1=V_nV_1$

P_2 =The remaining edges of G

The collection of graphoidal tree covering is $\mathcal{T}=\{P_1\cup P_2\}$

The graphoidal tree covering number of G is $\eta_T(G)=2$

Theorem 2.2

Let G be a bicyclic graph containing a $U(l,m)$.Let n denote the number of pendant vertices of G. Thenthe graphoidal tree covering number of G is $\eta_T(G)=2$

Proof:

Let G be a bicyclic graph with n pendant vertices.

Let $V(G)=\{C_o,V_1,V_2,...,V_n,U_1,U_2,...,U_n\}$, note that each V_i and U_i is attached with a tree.

$P_1 = V_1C_O \cup U_1C_O$

P_2 = The remaining edges of tree

The collection of graphoidal tree covering is $Ŧ = \{P_1 \cup P_2\}$

Therefore, the graphoidal tree covering number of G is $\eta_T(G) = 2$

Theorem 2.3

The graphoidal tree covering number of a triangular snake graph with 2n-1 vertices is defined by

$\eta_T(G) = n$.

Proof:

Let G be a triangular snake graph with 2n-1 vertices.

Let $V(G) = \{V_1, V_1^{'}, V_2, V_2^{'}, V_3, V_3^{'} ..., V_{n-1}, V_{n-1}^{'}, V_n\}$

The graphoidal tree paths are defined as

$P = V_1V_1^{'}V_2V_2^{'}V_3V_3^{'}, ..., V_{n-1}V_{n-1}^{'}V_n$

$P_i = V_iV_{i+1}\ , i = 1, 2, ..., n-1$

The collection of graphoidal tree covering is $Ŧ = P \cup P_i\ ,\ i = 1, 2, ..., n$

The graphoidal tree covering number of G is $\eta_T(G) = n$.

Theorem 2.4

The graphoidal tree covering number of adouble triangular snake graph with 2n-1 vertices is defined by $\eta_T(G) = n+1$

Proof:

Let G be a triangular snake graph with 2n-1 vertices.

Let $V(G) = \{V_1, V_1^{'}V_1^{''}, V_2, V_2^{'}V_2^{''}, V_3, V_3^{'}V_3^{''} ..., V_{n-1}, V_{n-1}^{'}V_{n-1}^{''}, V_n\}$

$P^{'} = \{V_1, V_1^{'}, V_2, V_2^{'}, V_3, V_3^{'} ..., V_{n-1}, V_{n-1}^{'}, V_n\}$

$P^{''} = \left\{ V_1, V_1^{''}, V_2, V_2^{''}, V_3, V_3^{''} ..., V_{n-1}, V_{n-1}^{''}, V_n \right\}$

$P_i = V_i V_{i+1}$, $i = 1, 2, ..., n-1$

The collection of graphoidal tree coverings is Ŧ $= P^{'} \cup P^{''} \cup P_i$, $i = 1, 2, ..., n$

The graphoidal tree covering number of G is $\eta_T(G) = n+1$

Theorem 2.5

The graphoidal tree covering number of a graph $G = (C_3, F_n)$ with 3n vertices is defined by

$\eta_T(G) = 12$

Proof:

Let $G = (C_3, F_n)$

Let $V(G) = \left\{ C_1V_2, C_1V_3, C_1V_4, C_1V_5, C_1V_6, C_1V_2^{'}, C_1V_3^{'}, C_1V_4^{'}, C_1V_5^{'} \right\}$

$P_i = C_1V_i$, $i = 2, 3, 4, 5, 6$

$Q_i = C_1V_i^{'}$, $i = 2, 3, 4, 5$

$Q^{'} = C_1C_2$

$P^{'} = V_5^{'}C_2$

$P^{''}$ = The remaining edges of G.

(ie) $P^{''} = G = \left\{ P_i \cup Q_i \cup Q^{'} \cup P^{'} \right\}$

The collection of graphoidal tree covering is Ŧ = $\left\{ P_i, Q_i, Q^{'}, P^{'}, P^{''} \right\}$

Hence, the graphoidaltreecovering number of G is $\eta_T(G) = 12$

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